The Large Numbers Hypothesis and Quantum Mechanics

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Abstract

In this paper, the suggested similarity between micro and macrocosmos is extended to quantum behavior, postulating that quantum mechanics, like general relativity and classical electrodynamics, is invariant under discrete scale transformations. This hypothesis leads to a large scale quantization of angular momenta. Using the scale factor $\Lambda \sim 10^{38}$, the corresponding quantum of action, obtained by scaling the Planck constant, is close to the Kerr limit for the spin of the universe – when this is considered as a huge rotating black-hole – and to the spin of Godel's universe, solution of Einstein equations of gravitation. Besides, we suggest the existence of another, intermediate, scale invariance, with scale factor $\lambda \sim 10^{19}$. With this factor we obtain, from Fermi's scale, the values for the gravitational radius and for the collapse proper-time of a typical black-hole, besides the Kerr limit value for its spin. It is shown that the mass-spin relations implied by the two referred scale transformations are in accordance with Muradian's Regge-like relations for galaxy clusters and stars. Impressive results are derived when we use a λ-scaled quantum approach to calculate the mean radii of planetary orbits in solar system. Finally, a possible explanation for the observed quantization of galactic redshifts is suggested, based on the large scale quantization conjecture.

1 Introduction

One of the oldest curious features of particles physics and cosmology is the possibility of obtaining cosmological large numbers, as mass (M), radius (R) and age (T) of our universe, scaling up the typical values of mass (m), size (r) and life-time (t) appearing in particles physics, by a scale factor $\Lambda \sim 10^{38-41}$.

This fact has led to important ideas and developments, as Dirac's hypothesis that cosmological parameters vary with the age of universe[1] and strong gravity[2−5], that tries to derive the hadron properties from a scaling down of gravitational theory, treating particles as black-hole type solutions.

The strong gravity approach can be based on the scale invariance of general relativity, also present in classical electrodynamics: the gravitational and electromagnetic equations are invariant under a scale transformation of time intervals and distances, provided we scale too the correspondent coupling factors. With this philosophy, we can think the universe as a self-similar structure, with the same physical laws appearing at different scales[5].[1]

Nevertheless, this beautiful picture of nature apparently breaks down for the quantum behavior of micro-cosmos: the introduction of Planck's constant defines a very particular scale at which, distinct from large scales, the quantum effects must be considered. In mathematical language, we can say that quantum equations, as Schroedinger and Dirac ones, are not scale invariant, due to the presence of h.

The main purpose of this paper is to explore the picture mentioned above, extending the scale invariance to quantum behavior. The price to pay, as just discussed, is the scaling of Planck's constant appearing in quantum equations, leading to the quantization of large structures, treated till now as classical ones. This procedure may seem to be rather speculative in character, but it leads to so impressive coincidences that we need ask ourselves what truth it contains.

A further purpose is to show that there seems to exist another, intermediate, scale of invariance besides that considered by the large numbers hypothesis. As will be shown, the new scaling leads from particles to typical stars and black holes, in the same way that the original scaling leads from particles to the observed universe.

[1]Some years ago, it was shown that strong gravity can be obtained as QCD approximation for the hadron IR region[6]. This makes us argue about the possibility of extending the scale invariance conjecture to QCD itself.

2 The large scale quantization

The cosmological quantities M, R, and T can be related to m, r and t through the scale relations

$$\frac{T}{t} = \frac{R}{r} = \left(\frac{M}{m}\right)^{\frac{1}{2}} = \Lambda \tag{1}$$

with $\Lambda \sim 10^{38-41}$. As mass, time, and length are all we need for constructing a complete system of units, relation (1) completely defines the scale transformation from particles's world to cosmological one.

With help of (1) we can, for instance, scale h in order to obtain the scale invariance of quantum equations. From a simple dimensional analysis we have

$$\frac{H}{h} = \Lambda^3 \tag{2}$$

leading to a scaled quantum of action given by $H \sim 10^{81}$ J.s, if we choose $\Lambda \sim 10^{38}$.

What is the meaning of this quantization? A possible answer is that the angular momentum of a rotating universe must be of the order of $H/2\pi \sim 10^{80}$ J.s.[2]

There is yet no conclusive evidence that universe rotates, although some speculations about indirect evidences can be made, as the rotation of galaxies and their clusters[7] and the intergalactic magnetic field[8,9]. But the important point here is that, *if* universe rotates, it should do that with angular momentum of the order of $H/2\pi$, close to Godel's spin – the spin value for the rotating cosmological solutions of Einstein's gravitational equations[7] – and to the Kerr limit for the spin of a rotating black-hole with mass of order 10^{50} kg. It is also important to note that this order of magnitude for universe's angular momentum is within the limits for global rotation obtained from the cosmic microwave background anisotropy[10] and is close to the value derived

[2]Relation (2) was already used by Caldirola, Pavisic, and Recami to obtain, from Planck's constant, the angular momentum of the rotating universe[3]. It is important to note that, choosing $\Lambda \sim 10^{38}$, we are relating cosmos with typical hadrons[5], what differs from the Dirac original conjecture (that uses $\Lambda \sim 10^{39}$).

from the observed rotation of the plane of polarization of cosmic electromagnetic radiation[11−13].

Besides, (1) and (2) fit well with Muradian's Regge-like relation for galaxies and clusters[9], $J = \hbar(M/m)^{3/2}$, where, here, M stands for the mass of the object under consideration. It is easy to see that this relation is in accordance with (1) and (2) when M is the mass of universe.

3 The intermediate scale invariance

We can infer another, intermediate, scale of quantization, related to the angular momenta of stars, which values concentrate around the order $H'/2\pi \sim 10^{42}$ J.s[9], close to the Kerr limit for a rotating black-hole with mass $M' \sim 10^{30}$ kg.

With these values we calculate the scale factor

$$\lambda \equiv \frac{R'}{r} = \frac{T'}{t} = \frac{H'}{h}\left(\frac{M'}{m}\right)^{-1} \sim 10^{19} \tag{3}$$

where the first equality comes naturally from the Lorentz invariance, while the second is obtained, again, from a simple dimensional analysis.

Besides, with these values for λ, M' and H', we can infer the relation

$$\lambda = \left(\frac{M'}{m}\right)^{\frac{1}{3}} = \left(\frac{H'}{h}\right)^{\frac{1}{4}} \tag{4}$$

which, together (3), completely defines the new scale transformation. Equation (4), on the other hand, is in accordance with Muradian's Regge-like relation for stars and planets[9], $J = \hbar(M/m)^{4/3}$.

From (3), we can estimate the values of the λ-scaled quantities R' and T' and try to find some physical meaning for them. From Fermi's scale we obtain $R' \sim 10^4$ m and $T' \sim 10^{-4}$ s. The first can be compared with the gravitational radius of a typical star: with $M' \sim 10^{30}$ kg it comes $r_g = 2GM'/c^2 \sim 10^3$ m. The second can be compared with the collapse proper-time of the star, $\tau \sim r_g/c \sim 10^{-5}$ s.

Let us try to understand why the scaled quanta of action is close to the Kerr limit for the angular momenta of rotating black-holes, in both (Λ and

λ) cases.

Equations (1)-(4) can be put together in the unified form

$$\frac{R_n}{r} = \left(\frac{M_n}{m}\right)^{\frac{1}{n}} = \left(\frac{H_n}{h}\right)^{\frac{1}{n+1}} \tag{5}$$

with $n = 2$ in the Λ-case and $n = 3$ in the λ-one. From dimensional analysis, we obtain for the corresponding gravitational constants

$$G_n = g\left(\frac{M_n}{m}\right)^{\frac{1}{n}-1} \tag{6}$$

where g is the strong gravity constant.

Equating the Kerr limit $J_n^{Kerr} = G_n M_n^2/c$ to $H_n/2\pi$ given by (5) and using (6), we arrive at the interesting result $gm^2/\hbar c = 1$. Thus, the coincidence between $H_n/2\pi$ and J_n^{Kerr} can be based on the fact that the strong structure constant is of order of unity. Or, reversing the thought, it shows that hadrons can be considered as maximally rotating black-holes.

It is important to note that the intermediate scale of length and time is equal to the geometrical average between Fermi's and cosmological scales. In fact, $(Rr)^{1/2} = r\Lambda^{1/2} = r\lambda = R'$. It is this fact that guarantees the uniqueness of the gravitational constant, no matter whether we are dealing with stars or clusters of galaxies. Indeed, if G' and G are the gravitational constants at, respectively, λ- and Λ-scales, we have, from (6), $G/G' = (G/g)(g/G') = \lambda^2/\Lambda = 1$.

4 The quantum approach for solar system

Up to now we have only considered orders of magnitude, which, alone, cannot provide a solid enough basis for the large scale quantization conjecture. Nevertheless, in a recent paper[14] Oliveira Neto (and, more recently, Agnese and Festa[15]) has presented impressive quantal results concerned to the solar system, in very good quantitative accordance with the observational data.[3]

[3]My thanks to M. Moret for advising me about Oliveira Neto's work and to a referee for calling my attention to the paper by Agnese and Festa.

For circular orbits, the Newtonian law of gravitation gives $v^2 = GM/r$, where v is the orbital velocity, M is the mass of Sun and r is the radius of the orbit. Substituting this equation in the λ-scaled Bohr quantization condition $L = mvr = nH'/2\pi$ (m is the mass of the planet), we have

$$r = \frac{n^2 H'^2}{4\pi^2 GMm^2} \tag{7}$$

Agnese and Festa[15] have fitted all the planetary orbits with the relation $r = n^2 r_1$, with $r_1 = 0.0439$ a.u. So, with $G = 6.67 \times 10^{-11}$ m^3/kg.s^2, $M = 1.99 \times 10^{30}$ kg and $m = 2.10 \times 10^{26}$ kg (the average mass of the planets of the solar system), we find, from (7), $H' = (4\pi^2 GMm^2 r_1)^{1/2} = 1.2 \times 10^{42}$ J.s, that is, the scaled quantum of action obtained, from (3) and (4), in the context of the intermediate scale invariance conjecture.

5 The redshift quantization

An statistical analysis of astronomical data has suggested the quantization of cosmic redshifts[16−21], a fact that has not been explained in the context of the standard cosmological model. For galaxies, the data has shown a step of quantization between $cz = 24$ km/s and $cz = 72$ km/s[16,17]; or, from another analysis, between $cz = 6,4 \times 10^3$ km/s and $cz = 1,28 \times 10^4$ km/s[18]. These results are also corroborated, at least on the qualitative level, by the observation that galaxies tend to cluster in sharp walls, leaving vast regions devoid of them[19]. We shall now try to establish a possible connection between such observations and the large scale quantization conjecture, with help of some natural assumptions.

If galaxies are considered as freely moving in a flat space-time, it is natural to assume a superior limit for their momenta given by Mc, where M is the mass of universe. This limitation of the space of momenta of galaxies leads, through the large scale uncertainty relations, to the quantization of their space-time, with a quantum of length given by $\Delta r \sim H/Mc$. Using $H \sim 10^{81}$ J.s and $M \sim 10^{50}$ kg, we arrive at $\Delta r \sim 10^{23}$ m, which corresponds to a velocity step given by $\Delta v \sim 100$ km/s.

6 Concluding remarks

Although the curious results shown in this paper, the large scale quantization, if genuine, needs a theoretical explanation. We are probably far away of such a theory, but some remarks can be made in this direction.

A possible explanation for the quantization of large structures could be based on an evolutional point of view: the quantal nature of the universe during its initial times (when it had Fermi's scale) has molded its – apparently quantized – nowadays large scale structure. This hypothesis may be resonable for the case of galaxies and clusters. But it is very improbably that intermediate structures like stars and the Solar System maintain the memory about the initial conditions. Besides, it would be necessary to explain the existence of two different scales of quantization, which does not seem to be very simple.

Another line of reasoning is to explain the various faces of large quantization in a fragmented way, in the context of different classical approachs. As examples, we can mention the "oscillating universe" models[22,23], introduced to explain the redshift periodicity of galaxies. Or Nottale's "quantum-mechanical" model for solar system[24], obtained as a diffusion process based on the chaotic character of the planetary orbits[25]. Though distinct from the approach presented here, Nottale's model also uses a scaled Planck constant of order 10^{42} J.s.[4]

Finally, what seems to be the more drastic philosophy: to see the universe, including its quantal behavior, as indeed self-similar and to incorporate this feature into any fundamental description of the physical world.

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